2005-2006 Problems

Maine Mathematics Science and Engineering Talent Search

Round 5 (2005-2006) **Grades 9-12**

Deadline: (Postmark) January 31th

Mail solutions to: MMSETS

P.O. Box 496

Orono ME 04473

NOTE: Please read and follow the GUIDELINES TO PREPARE SOLUTIONS (after the problems). Print out, complete and attach the cover sheet to your solution. The cover sheet is after the GUIDELINES

1.The number 17 is an especially interesting number. The cube of 17 is 4913. If you add the digits of 4013, the sum is 4+0+1+3 = 17. We got back the number we cubedFind out, what other numbers have this property - the sum of the digits of the cube of the number is the number itself ?

2. Write down any nine digit number which uses each of the digits 1, 2, 3, ..., 9 once only. Change the number by re-writing it with the very first digit as the units digit at the end and otherwise keeping the digits in the same order. For example 354218697 becomes 542186973. This is called a cyclic permutation of the digits. By now you will have two numbers written down. Repeat the cyclic permutation again and again writing down all the new numbers you obtain until you get back to your first number. Add up these nine numbers. Prove that, whatever number you chose originally, the total obtained in this way is the same.

3. Run a ribbon around the box so that it makes a complete loop with two parallel pieces of ribbon on the top (and on the bottom) of the box. The ribbon crosses every face once, except the top and bottom, which it crosses twice.

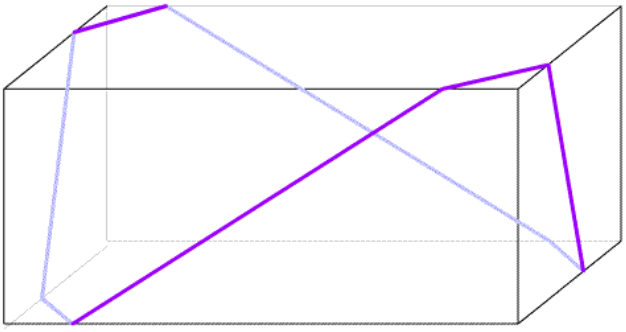
The ribbon rests tightly against the box all the way round because the angle at which it meets a corner is continued onto the next face.

Cut the ribbon in advance of placing it around the box and I can slide the ribbon around a little to position it.

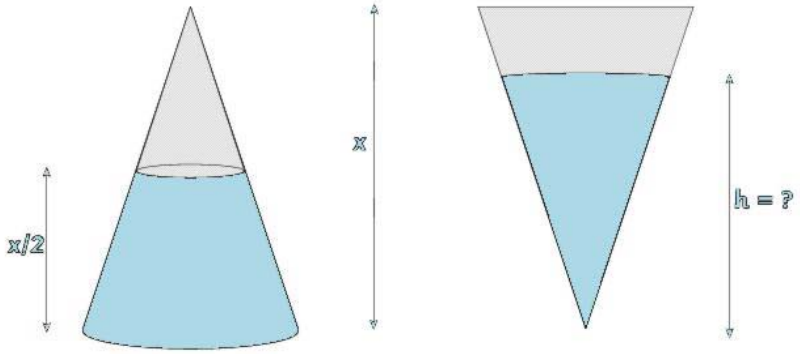
If the box is 20 in by 10in by 5in - how long will the ribbon be?

Show why it is possible to "slide" the ribbon.

What will it be for any box with height h, width w and length l? (n.b. the length and width are the longer distances and form the top of the box. Would the string be longer or shorter if this was not the case?)



4. The cone below is filled with water up to half of its height. If the cone is inverted, how high up the vertical height of the cone will the liquid rise?



5. The number 3723 in base 10 is written as 123 in another base. Could you find out what is that base ?

6. There are 78 prisoners in a square cell block of twelve cells. There is 1 prisoner in one of the cells, 2 in another cell, 3 in another, 4 in another and so on up to 12 prisoners in one of the cells. The clever prison warder made it easy to check if the

prisoners were all there by arranging them so there were 25 along each wall of the prison block. How did he do it?

7. Many modern codes are based on two very large prime numbers multiplied together.

This problem is based on a code using two different prime numbers less than 10. These two primes have been multiplied together and the resulting number has been used to shift the alphabet forward to new letters, assuming that A is at position 1, B at position 2 etc. For example, if the two prime numbers were 2 and 3, then to make the code, the alphabet would be shifted forward by 6 places. A would become G, B shifts to H and so on.

Which way will you need to shift the letters to decode?

When you have deciphered the code, there will be one word which will remain coded. You can decipher this word by adding the two prime numbers together and shifting the letters again.

8. Pick two cogs on the cogwheels below, place them near each other and turn one of them.
What happens?
Now pick a different pair and do the same thing again.

What do you notice about the direction in which the cogs turn each time?

Now pick two cogs with 5 "teeth".

9. Mark a "tooth" on each cog by selecting it and then pressing the blue button. You may like to line up the marked teeth so that they start next to each other.
Watch what happens to the teeth that are marked as the two cogs turn.
What can you say about the way that the marked tooth of one cog fits into the gaps of the second cog?

10. Will this always be the case whichever two cogs you pick?

Questions? Please e-mail to evaszillery-mmsets@me.acadia.net

Guidelines to prepare solutions

- Purpose: The purpose of organized writing is to help you explore and understand important mathematical concept. Written communication is key to comprehension: you can best understand mathematical ideas by explaining them clearly in writing.
- Write at a classmate's level: Specifically, write in such a manner that one of your classmates who are unfamiliar with the problem could easily follow your work. Thus, your solutions should be a well-organized, lucid explanation of what you're doing. In particular: clearly label all drawings and graphs. Identify any variables you use and, when appropriate, give their units. Don't pull formulas out of a hat (give a reference).
- Strike a balance between English sentences and mathematical equations: If your paper contains mathematical "chicken scratches" it will be almost impossible for a reader to follow what you have done.
- Make your paper presentable: Your paper need not be word-processed, but should be clean and neat. Don't scribble.
- Get an early start: many problems are challenging and require some experimentation. Starting a solution the night before it's due is a very bad idea.
- _ HYPERLINK "http://www.mmsets.org/Problem%20sets/mmsetscoversheet.doc" _Use our enclosed Cover Sheet_, fill out and attach to your solution set.
- Students in grade 6-9 can submit solutions to both problem sets (6-9 and/or 9-12).

Deadlines

In general, the participants of the MMSETS will have at least one month for the submission of their solutions. The deadlines for the seven rounds are:

Round 1 Due: October 4
Round 2 due November 1
Round 3 Due November 29
Round 4 due January 4 (Wednesday)
Round 5 January 31
Round 6 February 28
Round 7 March 28

These are the dates for postmarking the submissions; hence there is no need to utilize various special delivery services. We don't accept solutions submitted by e-mail.

The solutions submitted will be evaluated by a team of faculty members and graduate students. The scores will be sent to the students after each round. Except in cases of obvious oversights on our part, the scoring of the problems will be final. Appeals should be addressed to the coordinator of MMSETS.