

ARTIHMETIC MEAN, GEOMETRIC MEAN, HARMONIC MEAN  
and THE ROOT MEAN SQUARE

For two numbers,  $x$  and  $y$  we can have all sorts of means. We mention this here and now, as this information could be helpful in solving Problem #8 for grades 9-12.

There is the Arithmetic Mean (AM) =  $\frac{x+y}{2}$

The Geometric Mean (GM) =  $\sqrt{xy}$

And the Harmonic Mean (HM) =  $\frac{2}{\frac{1}{x} + \frac{1}{y}}$

We also have the root-mean square, which we shall call the Quadratic Mean

(QM) =  $\sqrt{\frac{x^2 + y^2}{2}}$  for two numbers.

The four means are related by the inequality

$$(QM) \geq (AM) \geq (GM) \geq (HM)$$

There is equality when  $x = y$