

# Logic

## Aristotelian Syllogistic: The Categorical Syllogism

### The Definition of a Syllogism

- “a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so” — Aristotle, *Prior Analytics* 24b18
- Looser & stricter definitions of the term “syllogism”
  - On Aristotle’s definition, all syllogisms are valid.
    - The proper structure (see below) is necessary to support a conclusion,
    - but not sufficient.
  - Does that mean it is improper to speak of an invalid syllogism?
    - Is it improper to speak of counterfeit money?
  - On more common modern definition:
    - the definition in terms of structure alone
    - then distinction between valid & invalid *syllogisms* is made

### The Necessary Structure of a Categorical Syllogism

- Components
  - Two premises & a conclusion
  - Three terms (exactly), each occurring in two distinct propositions
- Syllogism
  - “All mammals are animals; &
  - All horses are mammals.
  - So, All horses are animals.”
- But not
  - “All horses are mammals; &
  - All vertebrates are animals.
  - So, All horses are animals.” [Four Terms]
- Nor
  - “Some students who got D’s are mad; &
  - Anyone who’s mad should see a psychiatrist.
  - So, Some students who got D’s should see a psychiatrist.” [Four Terms!]

## Nomenclature

• Terms	
– Major term = the predicate of the conclusion	X m p
– Minor term = the subject of the conclusion	X s m
– Middle term = the term occurring in both premises (but not in the conclusion)	∴ X s p
• Propositions	
– Major premise = the premise containing the major term	X m p X s m
– Minor premise = the premise containing the minor term	∴ X s p

## Formal Aspects of the Categorical Syllogism

- There are two formal aspects of syllogisms necessary to determination of validity. (Recall that validity is a formal property.)
  - Mood—determined by logical form of constituent propositions
  - Figure—determined by placement of middle term

## The Mood of a Categorical Syllogism

- A list of the logical form of each proposition
  - by convention, the standard order is
    - Major Premise
    - Minor Premise
    - Conclusion
- Example
  - No mammals have gills.       E
  - All horses are mammals.     A
  - So, no horses have gills.    E
  - Mood—EAE

## The Figure of a Categorical Syllogism

- The three figures
  - Considering the predicate to be in some sense “broader than” its subject, the middle term may be

broader than the minor, narrower than the major	subject of the major, predicate of the minor	Amp <u>A</u> sm ∴.Asp	Figure 1
broader than both major & minor terms	predicate of both major & minor	Epm <u>A</u> sm ∴.Esp	Figure 2
broader than neither	subject of both major & minor	Amp <u>A</u> ms ∴.Isp	Figure 3

## How Many Valid Categorical Syllogisms Are There?

- There are many ways of counting
- The maximalist view
  - $4 \times 4 \times 4 = 64$  possible moods
  - 64 possible moods  $\times$  4 possible figures = 256 distinct syllogistic forms
    - Adding a fourth, “Galenic,” figure, in which the middle term is broader than (predicate of) the major & narrower than (subject of) the major
  - of these, 24 are valid syllogisms
- The minimalist view (Aristotle)
  - There are only a few valid moods in each of three figures
    - Fig. I has 4
    - Fig. II has 4
    - Fig. III has 6
    - Total: 14
- The difference is made up as follows
  - Fig. IV has 6 valid moods
  - Figs. I & II each have 2 additional moods in which a universal conclusion could be drawn from a premise pair, but a weaker, particular conclusion is pointlessly drawn instead.

# Figure I

Xmp  
Xsm  
∴.Xsp

Figure I: Principle

- The Scientific Figure
- Starting point—the *dictum de omni & nullo*
  - If every- or nothing of a certain kind [m] has a certain property [p], then
  - whatever [s] is of that kind [m]
  - [s] also has that [p] property
- Two paradigmatic syllogisms

All cows are mammals.	Acm	Barbara
All Holsteins are cows.	Ahc	
So, All Holsteins are mammals.	∴ Ahm	
No cows are birds.	Ecb	Celarent
All Holsteins are cows.	Ahc	
So, No Holsteins are birds.	∴ Ehb	

Figure I: Two More Valid Forms

- Weakening the minor premise allows a conclusion, but a weaker one:

All cows are mammals.	Acm	Darii
<i>Some farm animals</i> are cows.	Ifc	
So, <i>Some farm animals</i> are mammals.	∴ Ifm	
No cows are birds.	Ecb	Ferio
<i>Some farm animals</i> are cows.	Ihc	
So, <i>Some farm animals</i> are not birds.	∴ Ohb	

- This can be proven by adapting Fr Gensler's technique:

Proof of Darii

1. Acm
2. Ifc  
[∴ Ifm]
3. ~Ifm      assumption
4. Efm      3, Contradiction
5. Emf      4, Conversion
6. Ecf      5, 1, Celarent (right)
7. Efc      6, Conversion  
which contradicts 2
8. Ifm      discharging a-3. QED

5. Emf
1. <u>Acm</u>
∴ 6. Ecf

### Figure I: The Other Moods in Figure I

- So far, four premise pairs have been shown to yield conclusions in Figure I.
 

A	A	E	E
A	I	A	I
- The conclusions are as follows
  - AI & EI yield particular conclusions (I & O)
  - AA & EA yield universal conclusions (A & E)
  - Since universal conclusions can be weakened (by the Square of Opposition)
    - AA & EA would also yield particular conclusions (AAI & EAO)
    - but that's trivial & will be ignored.
- Note that the conclusion is always “dragged” down & to the right (on the Square of Opposition) by the premises:
  - If there is a particular premise, the conclusion must be particular.
  - If there is a negative premise, the conclusion must be negative.
  - These are general rules that apply to all figures.
- Aristotle shows that no other premise pairs yield conclusions in Figure I

### Aristotle's Proof that

#### Only four premise pairs have conclusions in Fig. I.

- He does this one premise pair at a time, as follows:
  - For each, he shows a true premise pair for which the “conclusion” is affirmative and another true premise pair for which it is negative.

All men are animals.	A	All men are animals.	A
No horses are men.	E	No stones are men.	E
So, All horses are animals.	A! = T	So, No stones are animals.	E! = T

- Since [Amp & Esm] as a premise pair is consistent with both Asp and Esp, neither can be inferred as a conclusion.
  - Weakening the conclusions gives both kinds of particular conclusion.

### Figure I: Restrictions

1. All valid first figure forms have a universal major
2. All valid first figure forms have an affirmative minor.
3. The conclusion of a first figure syllogism can be in any form.

Figure I: Rule-Case-Result Analysis

The Major Premise	states a rule	(a criterion about when to apply the major term) — Universal	All/No M are P.
The Minor Premise	presents a case that can be subsumed under the rule	(i.e., it says that the criterion holds of the minor term)— Affirmative	All/Some S are M
The Conclusion	states the result of applying the rule to the case	(i.e., it says how the major term is related to the minor)	any propositional form is possible

### Significance of RCR Analysis

- RCR Analysis explains the restrictions on first figure forms.
  - Rules are universal.
    - So the major premise must be universal.
  - Cases are affirmative.
    - So the minor premise must be affirmative.
  - Since the case might be universal or particular, the conclusion might be either.
  - Since the rule might be affirmative or negative, the conclusion might be either.
    - So, the conclusion might be in any form