The Stages of Aristotelian Logic

<table>
<thead>
<tr>
<th>Corresponding to ...</th>
<th>A Theory of ...</th>
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<tbody>
<tr>
<td>The First Act of the Intellect (Conception)</td>
<td>Terms</td>
</tr>
<tr>
<td>The Second Act of the Intellect (Judgment)</td>
<td>(Categorical) Propositions</td>
</tr>
<tr>
<td>The Third Act of the Intellect (Inference)</td>
<td>(Categorical) Syllogisms</td>
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</table>

“Just as in our minds, some thoughts are neither true nor false, while some are necessarily one or the other, so also in speech. For combination or division are necessary for falsity and truth.”—Aristotle, On Interpretation

Examples

- thoughts that are neither true nor false
  - dog, cat, tree
- thoughts that are necessarily true or false
  - dog are mammals; trees are plants

I. Terms in Categorical Propositions

- Singular
  - which name an individual person, thing, or event
  - examples
    - names (e.g., Socrates, the Peloponnesian War)
    - definite descriptions (e.g., my workman)
  - Singular terms can only occur as the subject of propositions
- Universal
  - which name kinds of things of which there could be many instances
  - examples
    - human beings
    - Athenians
    - people at Socrates’ trial
  - Universal terms can occur in propositions
    - either as the subject
    - or as the predicate

II. Propositional Forms

- A proposition is built from terms
  - by combination and division
  - The components are noun [phrase] and verb phrase
    - but here we will turn verb phrases back into noun phrases
    - “Silver is a horse” becomes “Silver + horse”
    - “Silver is white” becomes “Silver + white horse”
    - “Silver runs fast” becomes “Silver + horse that runs fast”
  - Not all sentences express propositions.
    - Statements do.
    - Questions, commands, &c., do not.
  - Proposition is a logical concept, not a linguistic one.
    - All these express the same proposition:
      - “Mark sees John”
      - “That boy [pointing to Mark] sees John”
      - “Marcus Ioannem vidit”
Propositions & Judgments

- The utterance of a proposition may or not express a judgment.
- A judgment is characterized by stating that a proposition is True or False.

Kinds of Propositions, I:
For Singular (i.e., Individual) Subjects

- Singular subjects
  - e.g., Socrates, Zeus
- Two initial kinds of propositions
  - combining the terms, e.g., Socrates is mortal.
  - separating the terms, e.g., Zeus is not mortal.
- Two kinds of judgment
  - affirming a proposition, e.g., “It is true that …”
  - denying a proposition, e.g., “It is false that …”
- This yields only two proposition types

<table>
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<th>Combination</th>
<th>Separation</th>
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<tr>
<td>Afirming...</td>
<td>Socrates is mortal.</td>
</tr>
<tr>
<td>Denying...</td>
<td>It is false that Socrates is mortal.</td>
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Kinds of Propositions, II:
For Universal Subjects

- Universal subjects
  - e.g., man, dog
- Two initial kinds of propositions
  - combining the terms, e.g., All men are mortal.
  - separating the terms, e.g., No gods are mortal.
- Two kinds of judgment
  - affirming a proposition, e.g., “It is true that …”
  - denying a proposition, e.g., “It is false that …”
- This yields four proposition types

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<td>Denying...</td>
<td>It is false that All men are mortal.</td>
</tr>
<tr>
<td>= Afirming a Separation: (but not the one above):</td>
<td>= Afirming a Combination: (but not the one above):</td>
</tr>
<tr>
<td>Some men are not mortal.</td>
<td>Some gods are mortal.</td>
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Four Kinds of Categorical Propositions
(when the Subject is Universal)

- These propositions are said to vary in two ways.
  - In quantity — universality or particularity
  - In quality — affirmativity or negativity

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<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Example</th>
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<tr>
<td>Universal Affirmative</td>
<td>A</td>
<td>All dogs are mammals.</td>
</tr>
<tr>
<td>Universal Negative</td>
<td>E</td>
<td>No dogs are mammals.</td>
</tr>
<tr>
<td>Particular Affirmative</td>
<td>I</td>
<td>Some dogs are mammals.</td>
</tr>
<tr>
<td>Particular Negative</td>
<td>O</td>
<td>Some gods are not mammals.</td>
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Analysis of Propositions

- Thesis: All propositions are categorical propositions
- Pro
  - All propositions have to be about something (= have a subject)
  - All propositions have to say something about that subject (= have a predicate)
  - So, all propositions have to have the form: S+P
  - All propositions have to affirm or deny the predicate of the subject (= have a quality)
  - All propositions have to say something about all or about some of the subject (= have a quantity)
- Con
  - Some propositions do not fit into this form (see next slide)

Three Kinds of Propositions

- The Three Kinds
  - Categorical Propositions
    - e.g., All dogs are mammals.
  - Existential Propositions
    - e.g., There is a God
  - Identity Statements
    - e.g., Clark Kent is Superman
- Comments
  - The non-categorical can be made to look categorical
    - e.g., God exists
  - Clark Kent is the person identical to Superman
  - But this raises problems
  - They can also be treated directly
Hard Cases I:
Cases where the predicate is not a noun phrase

- There are three possible kinds of (grammatical) predicate
  - predicate nouns (e.g., “John is a good football player.”)
  - predicate adjectives (e.g., “That horse is white.”)
  - verb phrases (e.g., “That horse runs well”)
- In the latter two, one must make the predicate into a term
  - “white” becomes “white horse”
  - “runs well” becomes “horse that runs well”

Hard Cases II:
Cases where the Quantifier is not Explicit

1. Singular subjects
   - e.g., “John is bright”, “Fred is not good at soccer”
   - Treat these as universals.
   - The whole point of the particular propositions is to emphasize that some part of what is named by the subject is outside of consideration;
   - this is not possible for propositions with singular subjects as there is only one thing named by the subject.
2. No quantifier at all
   - Contrast
     - “American military pilots must salute their superior officers”
     - “American military pilots bombed Serbian artillery positions yesterday”
   - Use common sense—the former is universal, the latter particular.
   - If you can’t tell, then either you lack sufficient knowledge of the subject or the author is being unclear.

Hard Cases III:
Cases where the Quality is Ambiguous

- “Some students are unpopular”
  - What’s the predicate?
    - “unpopular” with form I:
      - Some … are …: [students] + [unpopular]
    - “popular” with form O:
      - Some … are not …: [students] + [popular]
  - either analysis is possible if everyone is either popular or unpopular
    - Using [a] as the complement of a, or non-a:
      - Oab & Ia[b] (or Iab & Oa[b]) are equivalent expressions
    - Each is called the obverse of the other
    - (Obversion applies to universal propositions as well)

Obversion
- Obversion—a relation between propositions (or sentences)
  - Definition—the obverse of a proposition is one in which
    - (1) the subject of the obverse is the subject of the original,
    - (2) the predicate of the obverse is the complement of the predicate of the original
    - (3) the quality of the obverse is the opposite of the quality of the original
  - Examples
    - All horses are large hoofed mammals having a short-haired coat, a long mane, and a long tail.
    - No horses are not large hoofed mammals having a short-haired coat, a long mane, and a long tail.
    - Some horses are stallions.
    - Some horses are not non-stallions.
    - Some horses are not black.
    - Some horses are non-black.
    - No horses are persons.
    - All horses are non-persons.

Obversion (cont’d.)
- Logical Rules
  - The obverse of a true statement is always true.
  - The obverse of a false statement is always false.

Hard Cases IV:
Cases with Non-Standard Quantifiers

- “Not all…” is “Some … are not …” (O)
- “Most…”, “A few…” and their equivalents are particular (I or O)
- “Only A…” is “No non-A…”
  - e.g.,
    - “Only students who study logic diligently will do well” = “No students who don’t study logic diligently will do well”
Hard Cases IV:
Cases where cases there seems to be no Subject
• e.g., “It’s raining”; “There’s snow on the roads”
  – rephrase – “The weather is ...”; “The roads are snowy”

III. The Square of Opposition

Opposition
• What’s the opposite of “All dogs are brown”?
  – “No dogs are brown”?
  – “Some dogs are not brown”?
• There are two kinds of opposition
  – Contrary Opposition
    • Contrary opposites are a pair of propositions that (as a matter of logical necessity given their relation to one another) cannot both be true, but can both be false.
  – Contradictory Opposition
    • Contradictory opposites are a pair of propositions that (as a matter of logical necessity given their relation to one another) have opposite truth value.

The Square of Opposition
• The Square of Opposition shows the oppositions between propositions with the same subject and predicate but different logical form
  – A & E are contraries
    • “All dogs are brown” & “No dogs are brown”
  – A & O as well as E & I are contradictories
    • “All dogs are brown” & “Some dogs are not brown”
    • “No dogs are brown” & “Some dogs are brown”
  • It also shows the relation between
    – I & O, which are not contrary as they can both be true
      • “Some dogs are brown” & “Some dogs are not brown”
    – A & I as well as E & O
      • “All dogs are brown” & “Some dogs are brown”
      • “No dogs are brown” & “Some dogs are not brown”

Contradictory Opposition
• Definition
  – Contradictory opposites are a pair of propositions that (as a matter of logical necessity given their relation to one another) have opposite truth value.
• Place on the Square of Opposition
  – Propositions with the same subject & predicate, but different quantity & quality are contradictory opposites.
• Examples
  – The statements in the left & right column are equivalent.

<table>
<thead>
<tr>
<th>All horses are hoofed.</th>
<th>It’s false that some horses aren’t hoofed.</th>
</tr>
</thead>
<tbody>
<tr>
<td>No horses are fish.</td>
<td>It’s false that some horses are fish.</td>
</tr>
<tr>
<td>Some horses are black.</td>
<td>It’s false that no horses are black.</td>
</tr>
<tr>
<td>Some horses aren’t black.</td>
<td>It’s false that all horses are black.</td>
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</tbody>
</table>
Contrary Opposition

- **Definition**
  - Contrary opposites are a pair of propositions that (as a matter of logical necessity given their relation to one another) cannot both be true.
- **Place on the Square of Opposition**
  - Universal propositions with the same subject & predicate, but different quality are contrary opposites.
- **Examples**
  - The statements in the left column imply the statement in the right.
  - But those in the right do not imply those in the left.

<table>
<thead>
<tr>
<th>All horses are hooved.</th>
<th>It's false that no horses are hooved.</th>
</tr>
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<tbody>
<tr>
<td>No horses are fish.</td>
<td>It's false that all horses are fish.</td>
</tr>
</tbody>
</table>

Subcontrary Propositions

- **Subcontrary propositions are not really opposed to one another at all.**
  - They differ in quality, but they can both be true.
- **Definition (by place on the Square)**
  - Subcontrary propositions are a pair of particular propositions with the same subject & predicate, but opposite quality.
- **Logical Rule**
  - Subcontrary propositions cannot both be false.
- **Examples**
  - The statements in the left column imply the statement in the right.
  - But those in the right do not imply those in the left.

<table>
<thead>
<tr>
<th>It's false that some horses are blue.</th>
<th>Some horses are not blue.</th>
</tr>
</thead>
<tbody>
<tr>
<td>It's false that some horses are not mammals.</td>
<td>Some horses are mammals.</td>
</tr>
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Subalternation

- **This is even less a matter of opposition than was sub-contrariety.**
- **Weakening**
  - If a universal proposition is true, the particular proposition of the same quality with the same subject & predicate is also true.
  - Example
    - All dogs are mammals.
    - So, Some dogs are mammals.
  - Some people doubt this
    - But that is because they confuse two different things:
      - Logical implication, in which Asp implies Isp
      - Conversational implicature, in which one infers ~Asp from the fact that someone said Isp.
      - Presumably, if Asp had been true, the speaker would have said so.
      - But that presumption can be defeated.
- **Another inference**
  - If a particular proposition is false, the universal proposition of the same quality with the same subject & predicate is also false.
  - Example
    - It's false that some dogs are blue.
    - So, It's false that all dogs are blue.

IV. Conversion

The Definition of Immediate Inference

- Immediate inference is the direct inference of one proposition from another.
  - Immediate inference requires that the terms in the premise & the conclusion be the same (or be complements of one another).
  - Thus, they state the logical relations among categorical propositions with different but related subject & predicate.
  - The terms do not have to be in the same place in the two propositions,
    - i.e., the subject of the premise might be the predicate of the conclusion.
  - The quantity & quality of the premise & conclusion may or not be the same.

Kinds of Immediate Inference

- Obversion
- Conversion
- Contraposition
Obversion

- The obverse of a proposition is a proposition with changed quality & complemented predicate.
- E.g., Some dogs are brown, so some dogs are not non-brown.
- Always valid.
- See earlier comments.

Conversion

- The simple converse of a proposition is a proposition of the same quantity and quality as the original but with the subject and predicate reversed.
- Valid for E & I propositions.
  - No mammals are birds, so no birds are mammals.
  - Some dogs are brown, so some brown animals are dogs.
- For A propositions
  - not generally valid
    - All Saddam Hussein’s friends were opponents of the Gulf War. So all opponents of the war were friends of Saddam Hussein.
  - but valid for definitions
    - All men are rational animals, so all rational animals are men.
    - &valid after weakening (conversion per accidens)
      - All men are mammals, so some mammals are men.
- Not valid for O propositions
  - Some dogs are not collies, but not “some collies are not dogs.”

Contraposition

- The contrapositive of a proposition is the proposition with the same quantity & quality, but with the subject and predicate interchanged & complemented.
- Valid for A & O propositions (only)
  - All horses are large hoofed mammals having a short-haired coat, a long mane, and a long tail.
    - So, all animals that are not large hoofed mammals having a short-haired coat, a long mane, and a long tail are non-horses.
  - Some horses are not stallions
    - So, some non-stallions are not non-horses.
  - This is equivalent to obversion, conversion, & obversion again.
    - Some horses are not stallions.
    - So, some horses are non-stallions. [by obversion]
    - So, some non-stallions are horses. [by conversion]
    - So, some non-stallions are not non-horses. [by obversion]