PHIL 115:
Philosophical
Anthropology

Lecture #6:
Aristotelian Logic II:
Categorical Syllogisms

Overview

- Introduction to the Categorical Syllogism
- Validity in Categorical Syllogisms
  - By Figure
    - Figure 1: The Scientific Figure
    - Figure 2: The Figure of Separation
    - Figure 3: The Inductive Figure
  - Testing for Invalidity

Introduction to Categorical Syllogisms

Definitions

- Syllogism
  - “a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so”—Aristotle, Prior Analytics 24b18
- Categorical Syllogism
  - a syllogism in which the premises and conclusion are propositions beginning *all*, *some*, or *none*

The Necessary Structure of a Categorical Syllogism

- Components
  - Two premises & a conclusion
  - Three terms (exactly), each occurring in two distinct propositions
- Syllogism
  - “All mammals are animals; &
  - All horses are mammals.
  - So, All horses are animals.”
- But not
  - “All horses are mammals; &
  - All vertebrates are animals.
  - So, All horses are animals.”  [Four Terms]
- Nor
  - “Some students who got D’s are mad; &
  - Anyone who’s mad should see a psychiatrist.
  - So, Some students who got D’s should see a psychiatrist.”  [Four Terms]
Formal Aspects of
the Categorical Syllogism
• There are two formal aspects of syllogisms necessary to determination of
  validity. (Recall that validity is a formal property.)
  – Mood—determined by logical form of constituent propositions
  – Figure—determined by placement of middle term

The Mood of a
Categorical Syllogism
• A list of the logical form of each proposition
  – by convention, the standard order is
    • Major Premise
    • Minor Premise
    • Conclusion
  • Example
    • No mammals have gills. E
    • All horses are mammals. A
    • So, no horses have gills. E
  – Mood—EAE

The Figure of a
Categorical Syllogism
• The three figures
  – Considering the predicate to be in some sense “broader than” its subject,
    the middle term may be

<table>
<thead>
<tr>
<th>Broader than the minor, narrower than the major</th>
<th>Subject of the major, predicate of the minor</th>
<th>Amp, Acm, Aep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broader than both major &amp; minor terms</td>
<td>Predicate of both major &amp; minor</td>
<td>Epm, Apm, Esp</td>
</tr>
<tr>
<td>Broader than neither</td>
<td>Subject of both major &amp; minor</td>
<td>Amp, Amm, Isp</td>
</tr>
</tbody>
</table>

Figure 1: Principle

<table>
<thead>
<tr>
<th>All cows are mammals.</th>
<th>Acm</th>
<th>Barbara</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Holsteins are cows.</td>
<td>Acm</td>
<td></td>
</tr>
<tr>
<td>So, All Holsteins are mammals.</td>
<td>: Acm</td>
<td></td>
</tr>
<tr>
<td>No cows are birds.</td>
<td>Ech</td>
<td>Celarent</td>
</tr>
<tr>
<td>All Holsteins are cows.</td>
<td>Ech</td>
<td></td>
</tr>
<tr>
<td>So, No Holsteins are birds.</td>
<td>: Ech</td>
<td></td>
</tr>
</tbody>
</table>

Figure I

Xmp
Xsm
:: Xsp
Figure I: Two More Valid Forms

- Weakening the minor premise allows a conclusion, but a weaker one:

<table>
<thead>
<tr>
<th>Premise</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>All cows are mammals.</td>
<td>All Holsteins are mammals.</td>
</tr>
<tr>
<td>Some farm animals are cows.</td>
<td>Some farm animals are not birds.</td>
</tr>
<tr>
<td>So, Some farm animals are mammals.</td>
<td>So, Some farm animals are not mammals.</td>
</tr>
</tbody>
</table>

- The conclusion of a first figure syllogism can be in any form.

Figure I: The Trivial Valid Moods

- Any premise pair that yields a universal conclusion, will yield (via the Square of Opposition) a weakened version of that conclusion

<table>
<thead>
<tr>
<th>Barbara</th>
<th>Celarent</th>
</tr>
</thead>
<tbody>
<tr>
<td>All cows are mammals.</td>
<td>No cows are birds.</td>
</tr>
<tr>
<td>All Holsteins are cows.</td>
<td>All Holsteins are cows.</td>
</tr>
<tr>
<td>So, All Holsteins are mammals.</td>
<td>So, No Holsteins are birds.</td>
</tr>
<tr>
<td>All cows are mammals.</td>
<td>No cows are birds.</td>
</tr>
<tr>
<td>All Holsteins are cows.</td>
<td>All Holsteins are cows.</td>
</tr>
<tr>
<td>So, Some Holsteins are not birds.</td>
<td>So, Some Holsteins are not birds.</td>
</tr>
</tbody>
</table>

Figure I: Restrictions

1. All valid first figure forms have a universal major
2. All valid first figure forms have an affirmative minor.
3. The conclusion of a first figure syllogism can be in any form.

Figure II

- Are second figure syllogisms possible?

  - How can one relate two subjects by showing that they have a common predicate?
    - e.g., can one infer a relation between dogs & cats from the fact that both are mammals?—No.
  - but if the predicate were related to one subject and separated from the other, one could conclude that there is some separation between the two subjects
    - e.g., one can infer from the fact that all human beings can think and no computers can think that no human beings are computers.
  - The Exclusive Figure
    - Because the conclusion is always negative

- Figure II: Four Valid Moods

<table>
<thead>
<tr>
<th>Mood</th>
<th>Premise</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epm</td>
<td>All persons are capable of language use.</td>
<td>No computers are capable of common sense.</td>
</tr>
<tr>
<td>Apm</td>
<td>So, no computers are persons.</td>
<td>So, no human beings are computers.</td>
</tr>
<tr>
<td>Epm</td>
<td>Some contemporary executions are punishments not essential to the fight against crime.</td>
<td>No punishments that are not essential to the fight against crime are justified.</td>
</tr>
<tr>
<td>Apm</td>
<td>So, some contemporary executions are not justified.</td>
<td>So, some contemporary executions are justifiable.</td>
</tr>
<tr>
<td>Epm</td>
<td>All animals have the power of sensation.</td>
<td>No punishments that are not essential to the fight against crime are justified.</td>
</tr>
<tr>
<td>Apm</td>
<td>Some living things do not have the power of sensation.</td>
<td>So, some contemporary executions are not justified.</td>
</tr>
</tbody>
</table>
**Figure II: The Trivial Valid Moods**

- Any premise pair that yields a universal conclusion, will yield (via the Square of Opposition) a weakened version of that conclusion
  - These are not part of the fourteen basic valid syllogistic forms.

<table>
<thead>
<tr>
<th>Camestres</th>
<th>All cows are mammals.</th>
<th>All cows are mammals.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No crows are mammals.</td>
<td>No crows are mammals.</td>
</tr>
<tr>
<td></td>
<td>So, No crows are cows.</td>
<td>So, Some crows are not cows.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cesare</th>
<th>No crows are birds.</th>
<th>No crows are birds.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All crows are birds.</td>
<td>All crows are birds.</td>
</tr>
<tr>
<td></td>
<td>So, No crows are cows.</td>
<td>So, Some crows are not cows.</td>
</tr>
</tbody>
</table>

**Figure II: Restrictions**

1. The major premise must be universal
2. The minor premise can have any form
3. The conclusion must be negative

**Figure III**

\[ \text{Xmp} \]
\[ \text{Xms} \]
\[ \therefore \text{Xsp} \]

**Figure III: The Valid Moods**

<table>
<thead>
<tr>
<th>All lemons are sour.</th>
<th>Some human beings do stupid things.</th>
<th>All human beings are rational.</th>
</tr>
</thead>
<tbody>
<tr>
<td>All lemons are fruits.</td>
<td>So, some beings that do stupid things are rational beings.</td>
<td>No lemons are sweet.</td>
</tr>
<tr>
<td>So, Some fruits are sour.</td>
<td>So, some rational beings do stupid things.</td>
<td>All lemons are fruits.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Darapti</th>
<th>Disamis</th>
<th>Datisi</th>
<th>Felapton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amp Ans ; Isp</td>
<td>Imp Ans ; Isp</td>
<td>Amp Imo ; Isp</td>
<td>Emp Ans ; Osp</td>
</tr>
</tbody>
</table>

**Figure III: Restrictions**

1. Valid third figure syllogisms can have any form of major premise.
2. All valid third figure forms have an affirmative minor.
3. The conclusion of a third figure syllogism must be particular.

**Are third figure syllogisms possible?**

- How can one relate two predicates by showing that they have a common subject?
- If a subject has two predicates, the two predicates must be compatible
  - e.g., one can infer from the fact that all computers can process representations and all computers are machines that being a machine & being able to process representations are compatible, i.e., that some machines can process representations.
- The Inductive Figure
  - Because the subject of the premises can be considered a case on the basis of which a universal rule is refuted
Figure III as Refutation of a Rule by Counterexample

<table>
<thead>
<tr>
<th>SYLLOGISM</th>
<th>REFUTED RULE</th>
<th>COUNTEREXAMPLE (MIDDLE TERM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All lemons are sour. All lemons are fruits. Some fruits are sour.</td>
<td>No fruits are sour.</td>
<td>Lemons</td>
</tr>
<tr>
<td>Some human beings do stupid things. All human beings are rational. So, some rational beings do stupid things.</td>
<td>No rational beings do stupid things.</td>
<td>Human beings</td>
</tr>
<tr>
<td>All human beings are rational. Some human beings do stupid things. So, some beings that do stupid things are rational beings.</td>
<td>No beings that do stupid things are rational beings.</td>
<td>Human beings</td>
</tr>
<tr>
<td>No lemons are sweet. All lemons are fruits. So, some fruits are not sweet.</td>
<td>All fruits are sweet.</td>
<td>Lemons</td>
</tr>
<tr>
<td>Some citrus fruits are not sweet. All citrus fruits are fruit. So, some fruits are not sweet.</td>
<td>All fruits are sweet.</td>
<td>Citrus fruits</td>
</tr>
<tr>
<td>No living beings capable of local motion are plants. Some living beings capable of local motion are one-celled organisms. So, some one-celled organisms are not plants.</td>
<td>All one-celled organisms are plants.</td>
<td>Living beings capable of local motion</td>
</tr>
</tbody>
</table>

Testing Categorical Syllogisms for Validity

Invalidity by Counterexample

- The clearest way of showing that a syllogism is invalid is to produce a counterexample—an argument in that form (i.e., mood & figure) with true premises & a false conclusion.
  - E.g., for AAA 2d Figure
    Apm  All horses are four-legged.
    Asm  All cows are four-legged.
    Asp  So, all cows are horses.
- But that requires imagination.
  - There are less clear, but easier ways of knowing that a syllogism is invalid:
  - There are three sets of rules that all valid syllogisms must follow.

Requirements for Validity

- Constraints on the premises:
  - At least one premise must be universal.
  - At least one premise must be affirmative.
  - The middle term must be distributed at least once.
- Constraints on the conclusion:
  - One cannot get a universal conclusion from a particular premise.
  - One cannot get an affirmative conclusion from a negative premise
  - or a negative conclusion without a negative premise.
  - One cannot distribute in the conclusion what is not distributed in the premises.

Distribution of Terms

- In order to test syllogisms by rule, one needs the concept of distribution.
  - Terms are distributed in propositions when they refer in that proposition to all of the things the term names. (See next slide.)
  - More technically, when they can be narrowed without risk of making the proposition false. (See second next slide.)
- Distribution is part of a larger medieval theory of language.

What is Distributed?

- Terms are distributed in propositions when they refer in that proposition to all of the things the term names
- It is easy to see how this applies to the subjects of propositions:
  - Universal propositions distribute their subjects.
  - “All dogs are mammals” says something about all dogs.
  - As does “No dogs are mammals.”
  - Particular propositions do not distribute their subjects
  - “Some dogs are (or are not) collies” does not say something about all dogs.
- Less clear is its application to predicates:
  - Affirmative propositions do not distribute their predicates.
  - “All (or some) dogs are mammals” does not tell us anything about all mammals.
  - Negative propositions do distribute their predicates.
  - “No dogs are birds” tells us something about all birds, namely that they are not dogs.
  - What about “Some dogs are not collies”?
    - Think of one of the dogs that is not a collie, say Rin Tin Tin.
    - This proposition says of all collies that they are not identical to Rin Tin Tin.
What is Distributed? (cont’d.)

• A term is distributed in a certain kind of proposition if one can always narrow the term without changing a true proposition into a false one.

<table>
<thead>
<tr>
<th>Subject</th>
<th>Predicate</th>
</tr>
</thead>
<tbody>
<tr>
<td>All collies are dogs.</td>
<td>Some dogs are friendly animals.</td>
</tr>
<tr>
<td>Not true.</td>
<td>Not true.</td>
</tr>
<tr>
<td>Distributed.</td>
<td>Not distributed.</td>
</tr>
<tr>
<td>All big collies are dogs. &amp;c.</td>
<td>Some mean dogs are friendly (animals).</td>
</tr>
<tr>
<td>Not true.</td>
<td>Not true.</td>
</tr>
<tr>
<td>Distributed.</td>
<td>Not distributed.</td>
</tr>
<tr>
<td>All collies are ugly dogs.</td>
<td>Some dogs are not friendly (animals).</td>
</tr>
<tr>
<td>Not true.</td>
<td>Not true.</td>
</tr>
<tr>
<td>Not distributed.</td>
<td>Distributed.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distributions of the Subject</th>
<th>Distributions of the Predicate</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>A</td>
</tr>
<tr>
<td>O</td>
<td>I</td>
</tr>
</tbody>
</table>

Each propositional form has its own distribution pattern:
– E distributes both the Subject & the Predicate.
– A distributes the Subject only.
– O distributes the Predicate only.
– I distributes neither the Subject nor the Predicate.

Summary

Constructing Categorical Syllogisms

What kind of conclusion are you trying to prove? (A? E? I? O?)

<table>
<thead>
<tr>
<th>Fig. I</th>
<th>Fig. II</th>
<th>Fig. III</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A→p</td>
<td>A→p</td>
</tr>
<tr>
<td>E</td>
<td>E→p</td>
<td>E→p</td>
</tr>
<tr>
<td>I</td>
<td>I→p</td>
<td>I→p</td>
</tr>
<tr>
<td>O</td>
<td>O→p</td>
<td>O→p</td>
</tr>
</tbody>
</table>

Which Figure to Use?

• Any form of conclusion can be proven using a first figure syllogism
• A-propositions can only be proven in Fig. I
• E-propositions also in Fig. II (but not Fig. III)
• I-propositions also in Fig. III (but not Fig. II)
• O-propositions in any figure

Example Using First Figure

• Pattern
  – Conclusion: All S are P
  – What is it that makes them P
  – Because they are M & all M are P
• Example
  – To prove that a certain war is just
    • That war meets the criteria of the just-war theory
    • Any war that meets those criteria is just
  – To argue that embryonic stem-cell research is wrong
    • Embryonic stem-cell research includes the deliberate killing of innocent human beings
    • Any act that includes the deliberate killing of innocent human beings is wrong
For Refuting a Rule—Use Figure III

- Rule: “No wars ever lead to democracy”
- Refutation of that requires proof of—“Some wars lead to democracy”
- Counterexamples
  - World War II
- Argument
  - World War II led to democracy (in Germany & Japan).
  - World War II was a war.
  - So, some wars lead to democracy.