The Fall of the Rod

A long uniform rod of length L and mass M is pivoted about a horizontal, frictionless pin passing through one end of the rod. The rod is given a very slight push when it is in a vertical position.

At the instant the rod is horizontal find the following:

a) Find the angular speed.

b) Find the magnitude of the angular acceleration.

c) Find the horizontal and vertical components of acceleration of the center of mass.

d) Find the components of the reaction force at the pivot.

\[ \begin{align*}
\text{START: } & \text{ PE} \\
\text{END: } & \text{ rotational KE}
\end{align*} \]

\[ Mg h = \frac{1}{2} I \omega^2 \]

\[ h = \frac{L}{2} \]

\[ I = \frac{1}{3} ML^2 \]

(looked up)

a) \[ Mg \frac{L}{2} = \frac{1}{2} \left( \frac{1}{3} ML^2 \right) \omega^2 \]

\[ g = \frac{1}{3} L \omega^2 \Rightarrow \omega = \sqrt{\frac{3g}{L}} \]

b) Use \( \tau = I \alpha \) at instant rod is horizontal.

\[ \begin{align*}
\tau &= Mg \frac{L}{2} = \left( \frac{1}{3} ML^2 \right) \alpha \\
\Rightarrow \alpha &= \frac{3g}{2L}
\end{align*} \]

c) Horizontal component: Use the fact that this rod is moving in a circle. The sum of the forces in the \( \alpha \) direction should add up to \( ma = m \alpha \).
\[ \ddot{R} = M \frac{V^2}{r} \]

\[ a_x = \frac{V^2}{r} = \left( \frac{rw}{r} \right) = rw^2 = \frac{1}{2} \omega^2 \]

\[ = \frac{1}{2} \left( \sqrt{\frac{3g}{2}} \right)^2 = \frac{3g}{2} = a_x \]

**vertical component:**

\[ a_{y_{vert}} = \frac{g}{2} \left( \frac{3g}{2} \right) \]

\[ a_{y_{vert}} = \frac{3g}{4g} \text{ down} \]

**d)**

\[ \sum F_x = ma_{x_{vert}} \]

So \( x \) component of \( F_x = M \frac{3g}{2} \)

**horiz. comp.**

\[ \sum F_y = ma_{y_{vert}} \]

\[ F_y \]

\[ Mg - ( ) = M \frac{3g}{4g} \]

**vert. component of** \( F_n = \frac{1}{4} Mg \)