Circuits III

HOMEWORK


Note: First find the voltage across the 10 kΩ resistor as it's drawn in the circuit. Then find the voltage across that resistor when a 200 kΩ resistor (the non-ideal voltmeter) is in parallel with it. Then calculate the percent difference between the two measurements:

\[
\left| \frac{V_{\text{ideal}} - V_{\text{measured}}}{V_{\text{ideal}}} \right|
\]

**Without voltmeter**, \( R_{12} = (5+10) \, k\Omega = 15 \, k\Omega 

\[ I_{12} = \frac{E}{R_{\text{eq}}} = \frac{150\, V}{15 \, k\Omega} = 10 \, mA = I_1 = I_2 
\]

Because for both \( R_1 + R_2 

\[ V_2 = I_2 \cdot R_2 = (10\, mA) \cdot (10\, k\Omega) = 100\, V 
\]

\( \text{Ideal voltage} \)

\( \text{(Actual voltage when voltmeter not in place)} \)

**Now add non-ideal voltmeter:**

\[ \frac{R_1}{R_{2V}} \Rightarrow \frac{I_{2V}}{R_{\text{eq}}}, \quad \frac{1}{R_{2V}} = \frac{1}{R_2 + R_4} = \frac{1}{10} + \frac{1}{200} = \frac{21}{200} \quad \text{so} \quad R_{2V} = 9.052\, k\Omega 
\]

\[ R_{\text{eq}} = R_1 + R_{2V} = (5 + 9.52) \, k\Omega = 14.52 \, k\Omega 
\]

\[ I_{2V} = \frac{E}{R_{\text{eq}}} = \frac{150\, V}{14.52\, k\Omega} = 10.33\, mA = I_1 = I_2 
\]

\[ V_1 = I_2 \cdot R_1 = (10.33\, mA) \cdot (5\, k\Omega) = 51.65\, V, \quad V_2 = E - V_1 = 98.35\, V 
\]

\( \text{Measured voltage} \)

**Error:** \( \left| \frac{100 - 98.35}{100} \right| = 0.0165 \)

\( \text{or} \ 1.65\% \text{ error} \)

Note: First find the current through the 10 kΩ resistor as it's drawn in the circuit. Then find the current through that resistor when a 100 Ω resistor (the non-ideal ammeter) is in series with it. Then calculate the percent difference between the two measurements:

\[ \text{without ammeter, } R_{eq} = 15 \text{kΩ} \]

\[ I_{eq} = \frac{E}{R_{eq}} = \frac{150V}{15 \text{kΩ}} = 10 \text{mA} = I_1 = I_2 \]

\[ \text{with ammeter, } R_{eq} = 15.1 \text{kΩ} \]

\[ I_{eq} = \frac{E}{R_{eq}} = \frac{150V}{15.1 \text{kΩ}} = 9.934 \text{mA} \]

\[ \text{Error: } \left| \frac{10 - 9.934}{10} \right| \times 100\% = 0.662\% \]


\[ R_{eq} = (0.1 + 0.01)\Omega = 0.11 \text{Ω} \]

\[ I_{eq} = \frac{E}{R_{eq}} = \frac{12V}{0.11\Omega} = 109A \]

\[ \text{Huge! (many wires + meters cannot handle this)} \]

\[ \text{Power} = I_A^2 R_A = (109)^2 (0.1\Omega) = 1190 \text{Watts} \]

This will ruin the ammeter if it isn't protected by a fuse or circuit breaker.

(fuses are designed to break before the rest of the circuit is damaged)
Circuits III

4. This problem suggests one way to measure the emf and internal resistance of a real battery.

Imagine that you’ve wired the circuit shown in the figure. When switch $S$ is open, the (ideal) voltmeter reads 3.08 V. When the switch is closed, the voltmeter reading drops to 2.97 V, and the (ideal) ammeter reads 1.65 A. Find $E$, $r$, and $R$. Explain why the voltage drops when the switch is closed.

**Open switch**

No current flows anywhere (assuming resistance of $\text{V}$ is $\infty$ (ideal))

So voltmeter measures $E$ exactly $\Delta V = 3.08V$ (voltmeter)

Closed switch

Now current flows, and there is a voltage drop across the battery’s internal resistance

$\Delta V = E - IR \Rightarrow r = \frac{E - \Delta V}{I} = \frac{3.08 - 2.97}{1.65} = 0.0667\Omega$

This flows thru both $r$ and $R$

There is also a voltage drop across $R$, allowing us to determine its value:

$R = \frac{\Delta V}{I} = \frac{2.97}{1.65} = 1.80\Omega$

In summary, if switch is open, $\text{V}$ measures $E$. If switch is closed, $\text{V}$ measures $E - IR$, which is less than $E$. 
5. This figure shows the voltage across a capacitor that's charging through a 4700Ω resistor. Use the graph to determine (a) the battery voltage, (b) the time constant, and (c) the capacitance.

(a) \[ V_{\text{max}} = 9\text{ V} = V_B \]

(b) \[ \tau = \frac{RC}{V_C} = \frac{0.63V_B}{V_C} = 0.63 (9\text{ V}) = 5.7\text{ V} \]
\[ \tau \approx 1.5\text{ ms} \]

(c) \[ \tau = RC \]
\[ C = \frac{\tau}{R} = \frac{1.5 \times 10^{-3}}{4700\Omega} = 0.3\mu\text{F} \]

6. This simple circuit is called a "voltage divider" and has many useful applications. What value of \( R \) will make \( V_{\text{out}} = V_{\text{in}}/10 \)? Explain.

\[
\begin{align*}
V_{\text{out}} + V_R &= V_{\text{in}} \\
\frac{V_{\text{in}}}{10} + V_R &= V_{\text{in}} \\
V_R &= V_{\text{in}} \left(1 - \frac{1}{10}\right) \\
&= 9\frac{V_{\text{in}}}{10} \\
\frac{V_R}{V_{\text{in}}} &= \frac{9}{10} \text{ and } \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{10} \text{ (given)}
\end{align*}
\]

\[ \frac{9}{10} + \frac{1}{10} = \frac{10}{10} = 1 \]
\[ \text{So } R = 900\Omega \text{ (must } \neq 9R_{\text{out}}) \]
\[ \text{because both resistors share the same current} \]