**GROUP PROBLEM #8: GAUSS' LAW AND GAUSS' LAW WITH CONDUCTORS**

A point charge $Q$ is at the center of a spherical shell of radius $R$ carrying a charge $2Q$ spread uniformly across its surface. Write expressions for the electric field at $\frac{R}{4}$, $1.5R$, and $3R$.

At $r = \frac{R}{4}$:

$$E = \frac{kQ}{(\frac{R}{4})^2} = 16 \frac{kQ}{R^2}$$

At $r = 1.5R$:

$$E = \frac{k \cdot 3Q}{(\frac{3R}{2})^2} = \frac{25kQ}{3R^2}$$

At $r = 3R$:

$$E = \frac{kQ}{(3R)^2} = \frac{kQ}{3R^2}$$
You are helping to design a new electron microscope to investigate the structure of the HIV virus. A new device to position the electron beam consists of a charged circle of conductor. This circle is divided into two half circles separated by a thin insulator so that half of the circle can be charged positively and half can be charged negatively. The electron beam will go through the center of the circle. You have been asked to calculate the electric field if the positive charge on the half-circle is 2.5\(\text{nC}\), and the negative charge on the other half circle is -2.7\(\text{nC}\) and the radius of the circle is 2.5cm.

\[
\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- 
\]

\[
\mathbf{E}_+ = \frac{q_+}{A} = \frac{2.5 \times 10^{-9} \text{C}}{\pi R^2} = \frac{4\pi (9 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2) (2.5 \times 10^{-9} \text{C})}{\pi (0.025 \text{m})^2 + 2(0.025 \text{m})} = 2199.7 \text{N/C}
\]

\[
\mathbf{E}_- = \frac{q_-}{A} = \frac{-2.7 \times 10^{-9} \text{C}}{\pi R^2} = \frac{4\pi (9 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2) (-2.7 \times 10^{-9} \text{C})}{\pi (0.025 \text{m})^2 + 2(0.025 \text{m})} = -2375.6 \text{N/C}
\]

\[
\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = 2199.7 \text{N/C} - 2375.6 \text{N/C} = -175.93 \text{N/C}
\]
Find the electric field produced by a uniformly charged infinitely long wire carrying a charge of 87 pC/cm.

\[ E = \frac{\lambda}{2\pi r \varepsilon_0} \]

\[ E = \frac{87 \times 10^{-12} \text{ C}}{2\pi r \varepsilon_0} = \frac{2\pi \left(9 \times 10^6 \frac{\text{N m}^2}{\text{C}^2}\right)(87 \times 10^{-12} \text{ C})}{2\pi r} \]

\[ E = 1.566 \frac{\text{N}}{\text{C}} \]
A solid sphere with a radius of 10 cm carries 15 μC, distributed uniformly throughout the volume. Find the electric field strength at (a) \( r = 7 \text{ cm} \), (b) \( r = 15 \text{ cm} \), and (c) \( r = 23 \text{ cm} \).

\[
E = \frac{kQ}{r^2}
\]

\[
E_a = \frac{Q}{(4\pi \epsilon_0 R^2)^{\frac{1}{3}}} = \frac{Q}{R^3 \epsilon_0}
\]

\[
Q = 15 \times 10^{-6} \text{ C}
\]

\[
E_a = \frac{15 \times 10^{-6} \text{ C}}{0.01 \text{ m}^3}
\]

at \( r = 7 \text{ cm} = 0.07 \text{ m} \)

\[
E = \frac{Q}{(4\pi \epsilon_0 R^2)^{\frac{1}{3}}} = \frac{15 \times 10^{-6} \text{ C}}{0.07 \text{ m}^3}
\]

\[
E = \frac{15 \times 10^{-6} \text{ C}}{1.2 \times 10^8 \text{ N/C}} = \frac{1.25 \times 10^8 \text{ N/C}}{0.07 \text{ m}^3}
\]

at \( r = 15 \text{ cm} = 0.15 \text{ m} \)

\[
\left( \frac{r}{R} \right)^3 = 1
\]

\[
E = \frac{kQ}{r^2} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(15 \times 10^{-6} \text{ C})}{0.15 \text{ m}^2}
\]

\[
E = 6 \times 10^6 \text{ N/C}
\]

at \( r = 23 \text{ cm} = 0.23 \text{ m} \)

\[
\left( \frac{r}{R} \right)^3 = 1
\]

\[
E = \frac{kQ}{r^2} = \frac{(9 \times 10^9 \text{ Nm}^2/\text{C}^2)(15 \times 10^{-6} \text{ C})}{0.23 \text{ m}^2}
\]

\[
E = 2.35 \times 10^6 \text{ N/C}
\]