Review of Gravity (a lead-in to electric force)

1. For each figure, draw the net gravitational force vector, \( \vec{F}_{\text{net}} \), acting on each of the fixed masses. The lengths of your vectors should indicate the relative magnitudes of the forces. Or, label the mass with "\( \vec{F}_{\text{net}} = 0 \)" if no net force acts on it.

(a) \[ \begin{array}{c}
\vec{m} \\
\vec{m}
\end{array} \]

\[ \vec{F}_{12} = -\vec{F}_{21} \]

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(b) \[ \begin{array}{c}
5\vec{m} \\
\vec{m}
\end{array} \]

Magnitude is 5 times larger than in (a)

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(c) \[ \begin{array}{c}
\vec{m} \\
\vec{m} \\
\vec{m}
\end{array} \]

\[ \vec{F}_{\text{net}} = D \]

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(d) \[ \begin{array}{c}
\vec{m} \\
\vec{m}
\end{array} \]

\[ \begin{array}{c}
\vec{m} \\
\vec{m}
\end{array} \]
2. (a) Calculate the net gravitational force $\vec{F}_{\text{net}}$ acting on the 2m mass at the top of the figure. Write your answer in unit vector notation. Let $m = 3.2 \times 10^4$ kg. Be neat and organized so you can check your answer and so I can follow it.

(b) Then indicate the magnitude and direction of $\vec{F}_{\text{net}}$. Label units.

Draw an accurate sketch of $\vec{F}_{\text{net}}$.
3. Space explorers land on a planet that has the same mass as Earth, but find they weigh twice as much as they would on Earth. What’s the planet’s radius, given that the Earth radius is 6370 km?

\[
\text{Earth: weight} = \frac{GM_Em}{r_E^2}
\]

\[
\text{Planet: weight} = 2 \cdot \frac{GM_Em}{r_p^2} = \frac{GM_Em}{r_E^2} \Rightarrow \frac{r_p^2}{r_E^2} = \frac{1}{2} \Rightarrow r_p = \frac{r_E}{\sqrt{2}}
\]

\[
\text{Numerically: } r_p = \frac{6370 \text{ km}}{\sqrt{2}} = 4574 \text{ km}
\]

4. Two masses \(M\) and \(4M\) are separated by a distance \(D\). Determine the location \(x\) of a point \(P\) measured from \(M\) at which the net force on a third mass \(m_3\) would be zero.

At \(P\): \[
\vec{F}_{\text{net}} = \frac{Gmm_3}{x^2} \left( \frac{1}{(D-x)} \right) + \frac{G(4M)m_3}{(D-x)^2} \Rightarrow D
\]

Hence: \[
\frac{Gmm_3}{x^2} = \frac{4Gmm_3}{(D-x)^2} \Rightarrow 4x^2 = (D-x)^2 \Rightarrow 2x = D-x \Rightarrow D = 3x
\]

or \(x = \frac{1}{3} D \) (distance from \(m\))
5. You are in a spacecraft that is in a uniform circular orbit around the Moon, 130 km above the Moon’s surface. The radius of the Moon is 1740 km. Given that \( M_{\text{moon}} = 7.35 \times 10^{22} \text{ kg} \), what is your orbital period in hours? Hint: For linear motion, \( \vec{F}_{\text{tot}} = m\vec{a} \), but for circular motion \( \vec{F}_{\text{tot}} = ? \)

Hint: \( \vec{F}_{\text{tot}} = \frac{GM_{\text{moon}} m}{r^2} = \frac{mu^2}{r} \) where \( u = \frac{2\pi r}{T} \) is circumference of orbit

Then:\( \frac{GM_{\text{moon}}}{r^2} = \frac{u^2}{r} = \left( \frac{2\pi r}{T} \right)^2 = \frac{4\pi^2 r^3}{T^2} \Rightarrow T^2 = \frac{4\pi^2 r^3}{GM_{\text{moon}}} \)

Orbital period: \( T = 2\pi \sqrt{\frac{r^3}{6M_{\text{moon}}}} = 2\pi \sqrt{\frac{[1740(\text{km}) + 130(\text{km})]^3}{6 \times \frac{7.35 \times 10^{22}(\text{kg})}{\text{kg}^2}}} \)

\[ T = 7260 \text{ s} \times \frac{1 \text{ h}}{3600 \text{ s}} = 2.02 \text{ hours} \]