2) A thin glass rod is bent into a semicircle of radius $r$. A charge $+q$ is uniformly distributed along the upper half, and a charge $-q$ is uniformly distributed along the lower half, as shown in the figure. Find the magnitude and direction of the electric field at $P$, the center of the semicircle.

\[ \vec{E}_{\text{tot}} \text{ in } \uparrow \text{ direction} \]

Magnitude:

\[ E_{\text{tot}} = \int_{\text{upper arc}} \frac{k dq}{r^2} \cos \theta + \int_{\text{lower arc}} \frac{k dq}{r^2} \cos \theta \]

\[ E_{\text{tot}} = 2 \int_{\text{lower arc}} \frac{k dq}{r^2} \cos \theta = 2 \int_{\theta = 0}^{\pi/2} \frac{k \lambda r d\theta}{r^2} \cos \theta = \frac{2k\lambda}{r} \int_{0}^{\pi/2} \cos \theta d\theta = \frac{2k\lambda}{r} \left[ \sin \theta \right]_{0}^{\pi/2} = \frac{2k\lambda}{r} \]

Since $q = \int dq = \int_{0}^{\pi/2} \lambda r d\theta = \lambda r \theta \bigg|_{0}^{\pi/2} = \lambda r (\pi/2)$

we have $\lambda = \frac{q}{r} \left( \frac{2}{\pi} \right)$ and

\[ E_{\text{tot}} = \frac{2k \cdot \frac{q}{r} \cdot \frac{2}{\pi}}{\pi r^2} = \frac{4kq}{\pi r^2} \]

(Note: the negative sign in $-q$ was taken into account with the direction)