CAPACITOR PROBLEMS

Name: __________________________

1) Find the equivalent capacitance of the following systems. All of the capacitors have a capacitance of $C$.

\[
\begin{align*}
\frac{1}{C_{12}} &= \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{2} = \frac{2}{Z} \Rightarrow C_{12} = \frac{Z}{2} \\
C_{123} &= C_{12} + C_{34} = \frac{1}{2}C + \frac{1}{2}C = C
\end{align*}
\]

\[
\begin{align*}
C_{13} &= C_1 + C_3 = C + C = 2C \\
C_{123} &= C_{13} + C_{24} = 2C + 2C = 4C \\
\frac{1}{Z_{1234}} &= \frac{1}{C_{13}} + \frac{1}{C_{24}} = \frac{2}{2C} + \frac{2}{2C} = \frac{2}{Z} \Rightarrow Z_{1234} = \frac{Z}{2} = \frac{1}{C_{123}}
\end{align*}
\]

\[
\begin{align*}
C_{123} &= C_1 + C_2 + C_3 = C + C + C = 3C
\end{align*}
\]
3) Find the equivalent capacitance of the network. Find the total charge in the network. Find the charge on each capacitor. Find the voltage on each capacitor. Find the energy stored on each capacitor. Find the energy stored in the network.

\[ C_1 = 1.0 \mu F, \ C_2 = 3.0 \mu F, \ C_3 = 2.0 \mu F, \ \ V_{bat} = 30V \]

**Step 1:**

\[ C_{12} = C_1 + C_2 = 4 \mu F \]

**Step 2:**

\[ C_{123} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{4} + \frac{1}{2} = \frac{3}{4} \Rightarrow C_{123} = \frac{4}{3} \mu F \]

**Total charge:**

\[ q_{tot} = q_{eq} = C_{eq} V_{bat} = \left( \frac{4}{3} \mu F \right) (30V) = 40mC \]

For each capacitor \( \Rightarrow \) work backwards

\[ q_{tot} = q_3 = q_{12} = 40mC \]

\[ V_3 = \frac{q_3}{C_3} = \frac{40mC}{2mF} = 20V \]

\[ V_{12} = V_{bat} - V_3 = 30V - 20V = 10V = V_1 = V_2 \]

\[ q_1 = C_1 V_1 = (1mF)(10V) = 10mC \]

\[ q_2 = C_2 V_2 = (3mF)(10V) = 30mC \]

**Check:** \( q_1 + q_2 = q_{12} \)

**Energy stored:**

\[ U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} (1mF)(10V)^2 = 50mJ \]

\[ U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2} (3mF)(10V)^2 = 150mJ \]

\[ U_3 = \frac{1}{2} C_3 V_3^2 = \frac{1}{2} (2mF)(20V)^2 = 400mJ \]

\[ U_{tot} = 600mJ \]

\( (\Rightarrow \frac{1}{2} C_{eq} V_{bat}^2) \)